700+ GMAT Problem Solving Questions With Explanations

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1. Word problem / Mixture Problem
A container has 3L of pure wine. 1L from the container is taken out and 2L water is added.The process is repeated several times. After 19 such operations, qty of wine in mixture is
A. 2/7 L
B. 3/7 L
C. 6/19L

First operation: 3L-1L=2=**6/3L** of wine left, total 4L;
#2: 6/3L-(6/3)/4=6/3-6/12=18/12=**6/4L** of wine left, total 5L;
#3: 6/4L-(6/4)/5=6/4-6/20=24/20=**6/5L**, total 6L;
#4: 6/5L-(6/5)/6=6/5-6/30=30/30=**6/6L**, total 7L;
....

At this point it's already possible to see the pattern: x=6/(n+2)

n=19 --> x=6/(19+2)=6/21=2/7L

Answer: A.

Discussed at: http://gmatclub.com/forum/ratio-and-proportion-86312.html

2. Algebra / Modulus
What is the sum of all roots of the equation $|x+4|^2-10|x+4|=24$?

Solve for $|x+4|$ --> $|x+4|=12$ OR $|x+4|=-2$, BUT as absolute value never negative thus -2 is out. Solving $|x+4|=12$ --> $x_1=8$ or $x_2=-16$ --> $x_1+x_2=8-16=-8$.

Answer: -8.

Discussed at: http://gmatclub.com/forum/sum-of-all-roots-of-the-equation-85988.html#p645659

3. Algebra
If f(x)=5x^2 and g(x)=x^2 + 12x + 85, what is the sum of all values for k such that f (k+2)=g(2k)?

--> $k^2-4k-65=0$.

Viete's formula for the roots $x1$ and $x2$ of equation $ax^2+bx+c=0$:

$x1+x2=\frac{-b}{a}$ AND $x1*x2=\frac{c}{a}$

So in our case the roots $k1+k2=\frac{-(-4)}{1}=4$

Answer: 4.

Discussed at: http://gmatclub.com/forum/quadratic-querie-85989.html#p644602

4. Word problem
On a race track a maximum of 5 horses can race together at a time. There are a total of 25 horses. There is no way of timing the races. What is the minimum number of races we need to conduct to

get the top 3 fastest horses?
A. 5
B. 7
C. 8
D. 10
E. 11

First 5 races: all horses by five. We'll have the five winners.

Race 6: the winners of previous five races. We'll have the 3 winners.
Now it's obvious that #1 here is the fastest one (gold medal).
For the silver and bronze we'll have 5 pretenders:
1. #2 from the last sixth race,
2. #3 from the last sixth race,
3. the second one from the race with the Gold medal winner from the first five races,
4. the third one from the race with the Gold medal winner from the first five races,
5. the second one from the race with the one which took the silver in the sixth race

Race 7: these five horse: first and second in this one will have the silver and bronze among all 25.

Answer B.

Discussed at: http://gmatclub.com/forum/good-q-85080.html

5. Coordinate Geometry
In the rectangular coordinate system, points (4, 0) and (- 4, 0) both lie on circle C. What is the maximum possible value of the radius of C ?
A. 2
B. 4
C. 8
D. 16
E. None of the above

The only thing we can conclude from the question that center lies on the Y-axis. But it could be ANY point on it, hence we can not determine maximum value of r.

Answer: E.

Discussed at: http://gmatclub.com/forum/maximum-value-of-the-radius-86703.html

6. Word Problem
In an ensemble of gongs, all gongs have a diameter of either ten inches, or twelve inches or fifteen inches. In the collection there are 18 ten inch gongs. Half of the gongs in the collection are Tiger gongs. Of the Tiger gongs, there are equal numbers of ten inch, twelve inch and fifteen inch gongs. Half of the twelve inch gongs are not Tiger gongs, and half of all gongs are fifteen inches in diameter. How many gongs are there in the collection?
A. 18
B. 54
C. 72
D. 90
E. 108

If not the wording question won't be hard:

Let x and y be 12 and 15 inches gongs respectively. We know that ten inches are 18.

1. $18+x+y=S$. We want to calculate S.

2. "Half of the gongs in the collection are Tiger gongs" --> $2t=S$.

3. "Half of the twelve inch gongs are not Tiger gongs" --> means another half IS Tiger gongs, so x/2 is in Tiger gongs. As "Of the Tiger gongs, **there are equal numbers** of ten inch, twelve inch and fifteen inch gongs". --> x/2+x/2+x/2=t --> $\frac{3}{2}x=t$

4. "Half of all gongs are fifteen inches in diameter" --> $2y=S$

Four unknowns, four equations.

(3) $\frac{3}{2}x=t$ and (2) $2t=S$ --> $x=\frac{S}{3}$

(4) $2y=S$ --> $y=\frac{S}{2}$

(1) $18+x+y=S$ --> $18+\frac{S}{3}+\frac{S}{2}=S$ --> $S=108$

Answer: E.

Discussed at: http://gmatclub.com/forum/set-question-86769.html

7. Word Problems / Fractions
The rate of a certain chemical reaction is directly proportional to the square of the concentration of chemical A present and inversely proportional to the concentration of chemical B present. If the concentration of chemical B is increased by 100%, which of the following is closest to the percent change in the concentration of chemical A required to keep the reaction rate unchanged?
A. 100% decrease
B. 50% decrease
C. 40% decrease
D. 40% increase
E. 50% increase

NOTE: Put directly proportional in nominator and inversely proportional in denominator.
$RATE=\frac{A^2}{B}$, (well as it's not the exact fraction it should be multiplied by some constant but we can ignore this in our case).

We are told that B increased by 100%, hence in denominator we have 2B. We want the rate to be the same. As rate is directly proportional to the SQUARE of A, A should also increase (nominator) by x percent and increase of A in square should be 2. Which means x^2=2, x=~1.41, which is approximately 40% increase. $R=\frac{A^2}{B}=\frac{(1.4A)^2}{2B}$

Answer: D.

Discussed at: http://gmatclub.com/forum/ps-gmatprep-challenge-86954.html

8. Rate problem
Car B begins moving at 2 mph around a circular track with a radius of 10 miles. Ten hours later, Car A leaves from the same point in the opposite direction, traveling at 3 mph. For how many hours will Car B have been traveling when car A has passed and moved 12 miles beyond Car B?
A. 4pi - 1.6
B. 4pi + 8.4
C. 4pi + 10.4
D. 2pi - 1.6
E. 2pi - 0.8

It's possible to write the whole formula right away but I think it would be better to go step by step:

B speed: 2 mph;
A speed: 3 mph (travelling in the opposite direction);
Track distance: $2*\pi*r=20*\pi$;

What distance will cover B in 10h: $10*2=20$ miles
Distance between B and A by the time, A starts to travel: $20*\pi-20$

Time needed for A and B to meet distance between them divided by the relative speed: $\frac{20*\pi-20}{2+3}=\frac{20*\pi-20}{5}=4*\pi-4$, as they are travelling in opposite directions relative speed would be the sum of their rates;

Time needed for A to be 12 miles ahead of B: $\frac{12}{2+3}=2.4$;

So we have three period of times:
Time before A started travelling: 10 hours;
Time for A and B to meet: $4*\pi-4$ hours;
Time needed for A to be 12 miles ahead of B: 2.4 hours;

Total time: $10+4*\pi-4+2.4=4*\pi+8.4$ hours.

Answer: B.

Discussed at: http://gmatclub.com/forum/rates-on-a-circular-track-86675.html

9. Rate problem
A man cycling along the road noticed that every 12 minutes a bus overtakes him and every 4 minutes he meets an oncoming bus. If all buses and the cyclist move at a constant speed, what is the time interval between consecutive buses?
A. 5 minutes
B. 6 minutes
C. 8 minutes
D. 9 minutes
E. 10 minutes

Let's say the distance between the buses is d. We want to determine $Interval=\frac{d}{b}$, where b is the speed of bus.

Let the speed of cyclist be c.

Every 12 minutes a bus overtakes cyclist: $\frac{d}{b-c}=12$, $d=12b-12c$;

Every 4 minutes cyclist meets an oncoming bus: $\frac{d}{b+c}=4$, $d=4b+4c$;

$d=12b-12c=4b+4c$, --> $b=2c$, --> $d=12b-6b=6b$.

$Interval = \frac{d}{b} = \frac{6b}{b} = 6$

Answer: B.

Discussed at: http://gmatclub.com/forum/bus-86404.html

10. Modulus / Inequality

If $\frac{x}{|x|} < x$ which of the following must be true about x?

A. $x > 1$
B. $x > -1$
C. $|x| < 1$
D. $|x| = 1$
E. $|x|^2 > 1$

First of all let's solve this inequality step by step and see what is the solution for it, or in other words let's see in which ranges this inequality holds true.

Two cases for $\frac{x}{|x|} < x$:

A. $x < 0$ --> $|x| = -x$ --> $\frac{x}{-x} < x$ --> $-1 < x$ --> $-1 < x < 0$;

B. $x > 0$ --> $|x| = x$ --> $\frac{x}{x} < x$ --> $1 < x$.

So given inequality holds true in the ranges: $-1 < x < 0$ and $x > 1$. Which means that x can take values only from these ranges.

------{-1}xxxx{0}----{1}xxxxxx

Now, we are asked which of the following must be true about x. Option A can not be ALWAYS true because x can be from the range $-1 < x < 0$, eg $-\frac{1}{2}$ and $x = -\frac{1}{2} < 1$.

Only option which is ALWAYS true is B. ANY x from the ranges $-1 < x < 0$ and $x > 1$ will definitely be more the -1, all "red", possible x-es are to the right of -1, which means that all possible x-es are more than -1.

Answer: B.

Discussed at: http://gmatclub.com/forum/inequality-68886-20.html

11. Number Properties

A symmetric number of an another one is a number where the digit are reversed. for instance 123 is the symmetric of one of 321. Thus the different of a number and its symmetrical must be divisible by which of the following?

A. 4
B. 5
C. 6
D. 7
E. 9

Let's consider the example of three digit symmetric numbers {abc} and {cba}. Three digit number can be represented as: {abc}=100a+10b+c and {cba}=100c+10b+a. The difference would be:

{abc}-{cba}=100a+10b+c-(100c+10b+a)=99a-99c=99(a-c).

Two digit: {ab} and {ba}. {ab}-{ba}=10a+b-(10b+a)=9a-9b=9(a-b)

Hence the difference of two symmetric numbers (2 digit, 3 digit, ...) will always be divisible by 9.

Answer: E.

Discussed at: http://gmatclub.com/forum/number-properties-hard-question-helpp-89340.html

12. Number properties / Fractions
If 10*x/(x+y)+ 20*y/(x+y)=k and if x is less than y, which of the following could be the value of k?
A. 10
B. 12
C. 15
D. 18
E. 30

$$10*\frac{x}{x+y}+20*\frac{y}{x+y}=k$$

$$10*\frac{x+2y}{x+y}=k$$

$$10*(\frac{x+y}{x+y}+\frac{y}{x+y})=k$$

Finally we get: $10*(1+\frac{y}{x+y})=k$

We know that $x<y$

Hence $\frac{y}{x+y}$ is more than 0.5 and less than 1

$$0.5<\frac{y}{x+y}<1$$

So, $15<10*(1+\frac{y}{x+y})<20$

Only answer between 15 and 20 is 18.

There can be another approach:

We have: $\frac{10x+20y}{x+y}=k$, if you look at this equation you'll notice that it's a weighted average.

There are x red boxes and y blue boxes. Red box weight is 10kg and blue box weight 20kg, what is the average weight of x red boxes and y blue boxes?

k represents the weighted average. As y>x, then the weighted average k, must be closer to 20 than to 10. 18 is the only choice satisfying this condition.

Answer: D (18).

Discussed at: http://gmatclub.com/forum/must-be-an-easier-way-88620.html#p668499

13. Remainders
What is the remainder when (1!)^3+ (2!)^3 + (3!)^3 +.....(1152!)^3 is divided by 1152?
A. 125
B. 225
C. 325

We have the sum of many numbers: $(1!)^3+(2!)^3+(3!)^3+...+(1152!)^3$ and want to determine the remainder when this sum is divided by 1152.

First we should do the prime factorization of 1152: $1152=2^7*3^2$.

Consider the third and fourth terms:
$(3!)^3=2^3*3^3$ not divisible by 1152;
$(4!)^3=2^9*3^3=2^2*3*(2^7*3^2)=12*1152$ divisible by 1152, and all the other terms after will be divisible by 1152.

We'll get $\{(1!)^3+(2!)^3+(3!)^3\}+\{(4!)^3+...+(1152!)^3\}=225+1152k$ and this sum divided by 1152 will result remainder of 225.

Answer: A.

Discussed at: http://gmatclub.com/forum/numbers-86324.html

14. Number Properties
If X is a positive integer and 405^4 is a multiple of 3^X, what is the largest possible value of X?
A. 5.
B. 12.
C. 16.
D. 20.
E. 26

Given: $405^4=3^xk$ --> $405^4=(3^4*5)^4=3^{16}5^4=3^xk$. Hence the largest possible value of x is 16 (for $k=5^4$)

Answer: B.

Discussed at: http://gmatclub.com/forum/a-is-a-prime-number-a-91113.html

15. Geometry
A square wooden plaque has a square brass inlay in the center ,leaving a wooden strip of uniform width around the brass square.if the ratio of the brass area to the wooden area is 25 to 39,which of the following could be the width ,in inches ,of the wooden strip.
I. 1
II. 3
III. 4

A. I only
B. II only
C. III only
D. I and III only
E. I,II and III

Why would ANY width of the strip be impossible?

Let the the side of small square be x and the big square y.

Given: $\frac{x^2}{y^2-x^2}=\frac{25}{39}$ --> $\frac{x^2}{y^2}=\frac{25}{64}$ --> $\frac{x}{y}=\frac{5}{8}$.

We are asked which value of $\frac{y-x}{2}$ is possible. $\frac{y-\frac{5}{8}y}{2}=\frac{3}{16}y=?$.

Well, expression $\frac{3}{16}y$ can take ANY value depending on y : 1, 3, 4, 444, 67556, 0,9, ... ANY. Basically we are given the ratios of the sides (5/8), half of their difference can be any value we choose, there won't be any "impossible" values at all.

Discussed at: http://gmatclub.com/forum/hard-problem-og-quant-2nd-edition-89215.html#p674564

16. Statistics

If the mean of set S does not exceed mean of any subset of set S , which of the following must be true about set S ?

I. Set S contains only one element
II. All elements in set S are equal
III. The median of set S equals the mean of set S

A. none of the three qualities is necessary
B. II only
C. III only
D. II and III only
E. I, II, and III

"The mean of set S does not exceed mean of **any** subset of set S" --> set S can be:

A. $S=\{x\}$ - S contains only one element (eg {7});
B. $S=\{x,x,...\}$ - S contains more than one element and all elements are equal (eg{7,7,7,7}).

Why is that? Because if set S contains two (or more) different elements, then we can always consider the subset with smallest number and the mean of this subset (mean of subset=smallest number) will be less than mean of entire set (mean of full set>smallest number).

Example: S={3, 5} --> mean of S=4. Pick subset with smallest number s'={3} --> mean of s'=3 --> 3<4.

Now let's consider the statements:

I. Set S contains only one element - not always true, we can have scenario B too ($S=\{x,x,...\}$);

II. All elements in set S are equal - true for both A and B scenarios, hence always true;

III. The median of set S equals the mean of set S - - true for both A and B scenarios, hence always true.

So statements II and III are always true.

Answer: D.

Discussed at: http://gmatclub.com/forum/ps-challenge-93565.html

17. Number Problems
For every positive even integer n, the function h(n) is defined to be the product of all the even integers from2 to n, inclusive. If p is the smallest factor of h(100) + 1, then p is

A. between 2 and 10
B. between 20 and 10
C. between 20 and 30
D. between 30 and 40
E. greater than 40

$$h(100)+1=2*4*6*...*100+1=2^{50}*(1*2*3*..*50)+1=2^{50}*50!+1$$

Now, two numbers $h(100)=2^{50}*50!$ and $h(100)+1=2^{50}*50!+1$ are consecutive integers. Two consecutive integers are co-prime, which means that they don't share ANY common factor but 1. For example 20 and 21 are consecutive integers, thus only common factor they share is 1.

As $h(100)=2^{50}*50!$ has all numbers from 1 to 50 as its factors, according to above $h(100)+1=2^{50}*50!+1$ won't have ANY factor from 1 to 50. Hence p (>1), the smallest factor of $h(100)+1$ will be more than 50.

Answer: E.

Discussed at: http://gmatclub.com/forum/hard-question-function-and-factors-90863.html#p693396

18. Rate Problem
A bus from city M is travelling to city N at a constant speed while another bus is making the same journey in the opposite direction at the same constant speed. They meet in point P after driving for 2 hours. The following day the buses do the return trip at the same constant speed. One bus is delayed 24 minutes and the other leaves 36 minutes earlier. If they meet 24 miles from point P, what is the distance between the two cities?
A. 48
B. 72
C. 96
D. 120
E. 192

Distance between the cities d.

First meeting point $\frac{d}{2}$, as both buses travel at the same constant speed and leave the cities same time they meet at the halfway.

Total time to cover the d 4 hours, as the buses meet in 2 hours.

On the second day first bus traveled alone 1 hour (36min +24min), hence covered $0.25d$, and $0.75d$ is left cover.

They meet again at the halfway of $0.75d$, which is 24 miles from $\frac{d}{2}$:

$\frac{d}{2} - 24 = \frac{0.75d}{2}$

$d = 192$

Answer: E.

Discussed at: http://gmatclub.com/forum/two-buses-same-speed-head-spinning-86478.html

19. Number Properties

If n is multiple of 5, and $n = p^2q$ where p and q are prime, which of the following must be a multiple of 25?

A p^2

B. q^2

C. pq

D. p^2q^2

E. p^3q

$n = 5k$ and $n = p^2p$, (p and q are primes).

Q: $25m = ?$

Well obviously either p or q is 5. As we are asked to determine which choice MUST be multiple of 25, right answer choice must have BOTH, p and q in power of 2 or higher to **guarantee** the divisibility by 25. Only D offers this.

Answer: D.

Discussed at: http://gmatclub.com/forum/gmatprep-ps-questions-need-help-93922.html

20. Statistics

If d is the standard deviation x, y, and z, what is the standard deviation of x+5, y+5, z+5

A. d

B. 3d

C. 15d

D. d+5

E. d+15

TIP:

If we add or subtract a constant to each term in a set:

Mean will increase or decrease by the same constant.

SD will not change.

If we increase or decrease each term in a set by the same percent (multiply by a constant):

Mean will increase or decrease by the same percent.

SD will increase or decrease by the same percent.

So in our case SD won't change as we are adding 5 to each term in a set --> SD=d.

Answer: A.

Discussed at: http://gmatclub.com/forum/gmatprep-ps-questions-need-help-93922.html

21. Work Problem

It takes machine A 'x' hours to manufacture a deck of cards that machine B can manufacture in 'y' hours. If machine A operates alone for 'z' hours and is then joined by machine B until 100 decks are finished, for how long will the two machines operate simultaneously?

A. (100xy - z)/(x + y)
B. y(100x - z)/(x + y)
C. 100y(x - z)/(x + y)
D. (x + y)/(100xy - z)
E. (x + y - z)/100xy

Note that we are asked: "for how long will the **two machines** operate **simultaneously**?".

In first z hours machine A alone will manufacture $\frac{z}{x}$ decks. So there are $100-\frac{z}{x}=\frac{100x-z}{x}$ decks left to manufacture. Combined rate of machines A and B would be $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$ decks/hour, (remember we can easily sum the rates).

As $time=\frac{job}{rate}$, then $time=\frac{100x-z}{x}*\frac{xy}{x+y}=\frac{y(100x-z)}{x+y}$.

Answer: B.

Discussed at: http://gmatclub.com/forum/deck-of-cards-95608.html

22. Fractions
Every day a certain bank calculates its average daily deposit for that calendar month up to and including that day. If on a randomly chosen day in June the sum of all deposits up to and including that day is a prime integer greater than 100, what is the probability that the average daily deposit up to and including that day contains fewer than 5 decimal places?
A. 1/10
B. 2/15
C. 4/15
D. 3/10
E. 11/30

Theory:
Reduced fraction $\frac{a}{b}$ (meaning that fraction is already reduced to its lowest term) can be expressed as terminating decimal *if and only* b (denominator) is of the form 2^n5^m, where m and n are non-negative integers. For example: $\frac{7}{250}$ is a terminating decimal 0.028, as 250 (denominator) equals to $2*5^2$. Fraction $\frac{3}{30}$ is also a terminating decimal, as $\frac{3}{30}=\frac{1}{10}$ and denominator $10=2*5$.

Note that if denominator already has only 2-s and/or 5-s then it doesn't matter whether the fraction is reduced or not.

For example $\frac{x}{2^n5^m}$, (where x, n and m are integers) will always be terminating decimal.

(We need reducing in case when we have the prime in denominator other then 2 or 5 to see whether it could be reduced. For example fraction $\frac{6}{15}$ has 3 as prime in denominator and we need to know if it can be reduced.)

BACK TOT THE ORIGINAL QUESTION:

Question: does $average=\frac{p}{d}$ has less than 5 decimal places? Where $p=prime>100$ and d is the chosen day.

If the chosen day, d, is NOT of a type 2^n5^m (where n and m are nonnegative integers) then $average=\frac{p}{d}$ will not be a terminating decimal and thus will have more than 5 decimal places.

How many such days are there of a type 2^n5^m : 1, 2, 4, 5, 8, 10, 16, 20, 25 ($1=2^0*5^0$, $2=2^2$, $4=2^2$, 5, $8=2^3$, $10=2*5$, $16=2^4$, $20=2^2*5$, $25=5^2$), total of 9 such days (1st of June, 4th of June, ...).

Now, does p divided by any of these $d's$ have fewer than 5 decimal places? Yes, as $\frac{p}{d}*10{,}000=integer$ for any such d (10,000 is divisible by all these numbers: 1, 2, 4, 5, 8, 10, 16, 20, 25).

So, there are 9 such days out of 30 in June: $P=\frac{9}{30}=\frac{3}{10}$.

Answer: D.

Discussed at: http://gmatclub.com/forum/mgmat-challenge-decimals-on-deposit-97456.html

23. Sequences

What is the sixtieth term in the following sequence? 1, 2, 4, 7, 11, 16, 22...

A. 1,671
B. 1,760
C. 1,761
D. 1,771
E. 1,821

First we should find out what is the pattern of this sequence. Write the terms of the sequence as below:

$a_1=1$;
$a_2=2=a_1+1$;
$a_3=4=a_1+1+2$;
$a_4=7=a_1+1+2+3$;
$a_5=11=a_1+1+2+3+4$;
$a_6=16=a_1+1+2+3+4+5$;
$a_7=22=a_1+1+2+3+4+5+6$;
...

So the n_{th} term of this sequence equals to first term *plus* the sum of first $n-1$ integers.

$$a_{60}=a_1+1+2+3+...+59=a_1+\frac{1+59}{2}*59=1771.$$

Answer: D.

Discussed at: http://gmatclub.com/forum/help-tough-problem-on-exponential-sequence-97907.html

24. Powers / Number Properties

If 3^(6x) = 8,100, what is the value of [3^(x - 1)]^3 ?

A. 90
B. 30
C. 10
D. 10/3
E. 10/9

If $3^{6x}=8{,}100$, what is the value of $(3^{x-1})^3$? (It's not hard at all).

$3^{6x}=(3^{3x})^2=90^2=8{,}100$ --> $3^{3x}=90$.

$(3^{x-1})^3=3^{3x-3}=\frac{3^{3x}}{3^3}=\frac{90}{27}=\frac{10}{3}$.

Answer: D.

Discussed at: http://gmatclub.com/forum/anything-wrong-in-this-problem-can-anyone-dare-to-solve-98777.html

25. Inequalities / Algebra
If x and y are integers such that (x+1)^2 less than equal to 36 and (y-1)^2 less than 64. What is the largest possible and minimum possible value of xy.

$(x+1)^2\leq 36$ --> $-\sqrt{36}\leq x+1\leq \sqrt{36}$ --> $-6\leq x+1\leq 6$ --> $-7\leq x\leq 5$.

$(y-1)^2<64$ --> $-\sqrt{64}<y-1<\sqrt{64}$ --> $-8<y-1<8$ --> $-7<y<9$, as y is an integer we can rewrite this inequality as $-6\leq y\leq 8$.

We should try extreme values of x and y to obtain min and max values of xy:

Min possible value of xy is for $x=-7$ and $y=8$ --> $xy=-56$;
Max possible value of xy is for $x=-7$ and $y=-6$ --> $xy=42$.

Solving with absolute values gives the same results:

$(x+1)^2\leq 36$ means $|x+1|\leq 6$ --> $-7\leq x\leq 5$.
$(y-1)^2<64$ means $|y-1|<8$ --> $-7<y<9$.

Discussed at: http://gmatclub.com/forum/in-equalities-how-to-handle-an-expression-with-squares-97992.html

26. Powers
The operation x#n for all positive integers greater than 1 is defined in the following manner:
x#n = x to the power of x#(n-1)
If x#1 = x, which of the following expressions has the greatest value?
A. (3#2)#2
B. 3#(1#3)
C. (2#3)#2
D. 2#(2#3)
E. (2#2)#3

Couple of things before solving:

If exponentiation is indicated by stacked symbols, the rule is to work from the top down, thus:
$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$, which on the other hand equals to a^{mn}.

So:
$(a^m)^n=a^{mn}$;

$a^{m^n} = a^{(m^n)}$ and not $(a^m)^n$.

Back to the original question:
Let's replace # by @ as # looks like the symbol "not equal to" and it might confuse someone.

$x@n = x^{(x@(n-1))}$ and $x@1 = x$:

$x@2 = x^{(x@1)} = x^x$, as $x@1 = x$;
$x@3 = x^{(x@2)} = x^{(x^x)} = x^{x^x}$;
$x@4 = x^{(x@3)} = x^{(x^{x^x})} = x^{x^{x^x}}$;
...
Basically n in x@n represents the # of stacked x-es.

A. $(3@2)@2 = (3^3)@2 = (27)@2 = 27^{27} = 3^{81}$

B. $3@(1@3) = 3@(1^{1^1}) = 3@1 = 3$

C. $(2@3)@2 = (2^{2^2})@2 = 16@2 = 16^{16} = 2^{64}$

D. $2@(2@3) = 2@(2^{2^2}) = 2@16$ this will be huge number
2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2=2^2^2^2^2^2^2^2^2^2^2^2^2^2^4=2^2^2^2^2^2^2^2^2^2^2^2^16=.... ;

E. $(2@2)@3 = (2^2)@3 = 4@3 = 4^{4^4} = 4^{256} = 2^{512}$

2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2 will be much bigger number than numbers from other answer choices.

Answer: D.

Discussed at: http://gmatclub.com/forum/just-800-level-question-99064.html

27. Overlapping Sets
3/8 of all students at Social High are in all three of the following clubs: Albanian, Bardic, and Checkmate. 1/2 of all students are in Albanian, 5/8 are in Bardic, and 3/4 are in Checkmate. If every student is in at least one club, what fraction of the student body is in exactly 2 clubs?
A. 1/8
B. 1/4
C. 3/8
D. 1/2
E. 5/8

Let's # of students at Social High be 8 (I picked 8 as in this case 3/8 of total and 5/8 of total will be an integer).

3/8 of all students at Social High are in all three clubs --> 3/8*8=3 people are in exactly 3 clubs;

1/2 of all students are in Albanian club --> 1/2*8=4 people are in Albanian club;
5/8 of all students are in Bardic club --> 5/8*8=5 people are in Bardic club;
3/4 of all students are in Checkmate club --> 3/4*8=6 people are in Checkmate club;

Also as every student is in at least one club then # of students in neither of clubs is 0;

Total=A+B+C-{# of students in exactly 2 clubs}-2*{# of students in exactly 3 clubs}+{# of students in neither of clubs};

8=4+5+6-{# of students in exactly 2 clubs}-2*3+0 --> **{# of students in exactly 2 clubs}=1**, so fraction is 1/8.

Answer: A.

Discussed at: http://gmatclub.com/forum/the-quest-for-700-weekly-gmat-challenge-99549.html

28. Word Problem
A certain city with population of 132,000 is to be divided into 11 voting districts, and no district is to have a population that is more than 10 percent greater than the population of any other district. What is the minimum possible population that the least populated district could have?
A. 10,700
B. 10,800
C. 10,900
D. 11,000
E. 11,100

As "no district is to have a population that is more than 10 percent greater than the population of **any** other district", then the districts can have only two population #: x and $1.1x$.

So we want to minimize x. To minimize x we should make **only one** district to have that # of population (minimum possible) and the **rest 10** districts to have $1.1x$ # of population (maximum possible).

$x+10*1.1x=132$ --> $12x=132$ --> $x=11$.

Answer: D.

Discussed at: http://gmatclub.com/forum/gmat-prep-ps-93369.html

29. Remainders
What is the remainder when (18^22)^10 is divided by 7 ?
A 1
B 2
C 3
D 4
E 5

I think this question is beyond the GMAT scope. It can be solved with Fermat's little theorem, **which is not tested on GMAT**. Or another way:

$(18^{22})^{10}=18^{220}=(14+4)^{220}$ now if we expand this all terms but the last one will have 14 as multiple and thus will be divisible by 7. The last term will be 4^{220}. So we should find the remainder when 4^{220} is divided by 7.

$4^{220}=2^{440}$.

2^1 divided by 7 yields remainder of 2;
2^2 divided by 7 yields remainder of 4;
2^3 divided by 7 yields remainder of 1;

2^4 divided by 7 yields remainder of 2;
2^5 divided by 7 yields remainder of 4;
2^6 divided by 7 yields remainder of 1;
...

So the remainder repeats the pattern of 3: 2-4-1. So the remainder of 2^{440} divided by 7 would be the same as 2^2 divided by 7 (440=146*3+2). 2^2 divided by 7 yields remainder of 4.

Answer: D.

Discussed at: http://gmatclub.com/forum/remainder-99724.html

30. Number Properties
For any positive integer n, the length of n is defined as the number of prime factors whose product is n. For example; the length of 75 is 3, since 75= 3*5*5. How many two digit positive integers have the length 6?
A. None
B. One
C. Two
D. Three
E. Four

Basically the length of the integer is the sum of the powers of its prime factors.

Length of six means that the sum of the powers of primes of the integer (two digit) must be 6. First we can conclude that 5 can not be a factor of this integer as **the smallest integer with the length of six** that has 5 as prime factor is $2^5*5=160$ (length=5+1=6), not a two digit integer.

The above means that the primes of the two digit integers we are looking for can be only 2 and/or 3. $n=2^p*3^q$, $p+q=6$ max value of p and q is 6.

Let's start with the highest value of p:
$n=2^6*3^0=64$ (length=6+0=6);
$n=2^5*3^1=96$ (length=5+1=6);

$n=2^4*3^2=144$ (length=4+2=6) not good as 144 is a three digit integer.

With this approach we see that actually $5<=p<=6$.

Answer: B.

Discussed at: http://gmatclub.com/forum/700-algrbra-need-help-again-thanks-so-much-90320.html

31. Statistics
E is a collection of four *ODD* integers and the greatest difference between any two integers in E is 4. The standard deviation of E must be one of how many numbers?
A 3
B 4
C 5
D 6
E 7

Let the smallest odd integer be 1, thus the largest one will be 5. We can have following 6 types of sets:

1. {1, 1, 1, 5} --> mean=2 --> |mean-x|=(1, 1, 1, 3);
2. {1, 1, 3, 5} --> mean=2.5 --> |mean-x|=(1.5, 1.5, 0.5, 2.5);
3. {1, 1, 5, 5} --> mean=3 --> |mean-x|=(2, 2, 2, 2);
4. {1, 3, 3, 5} --> mean=3 --> |mean-x|=(2, 0, 0, 2);
5. {1, 3, 5, 5} --> mean=3.5 --> |mean-x|=(2.5, 0.5, 1.5, 1.5);
6. {1, 5, 5, 5} --> mean=4 --> |mean-x|=(3, 1, 1, 1).

CALCULATING STANDARD DEVIATION OF A SET {x1, x2, ... xn}:
1. Find the mean, m, of the values.

2. For each value x_i calculate its deviation ($m-x_i$) from the mean.
3. Calculate the squares of these deviations.
4. Find the mean of the squared deviations. This quantity is the variance.
5. Take the square root of the variance. The quantity is th SD.

Expressed by formula: $standard\ deviation = \sqrt{variance} = \sqrt{\frac{\sum(m-x_i)^2}{N}}$.

You can see that deviation from the mean for 2 pairs of the set is the same, which means that SD of 1 and 6 will be the same and SD of 2 and 5 also will be the same. So SD of such set can take only 4 values.

Answer: B.

Discussed at: http://gmatclub.com/forum/hard-standard-deviation-99774.html

32. Geometry

The dimensions of a rectangular solid are 4 inches, 5 inches, and 8 inches. If a cube, a side of which is equal to one of the dimensions of the rectangular solid, is placed entirely within the sphere just large enough to hold the cube, what the ratio of the volume of the cube to the volume within the sphere that is not occupied by the cube?

A. 10:17
B. 2:5
C. 5:16
D. 25:7
E. 32:25

The cube must have a side of 4 inches to fit in rectangular solid. Now as the cube is inscribed in sphere then the radius of this sphere equals to half of the cube's diagonal --> $d=\sqrt{4^2+4^2+4^2}=4\sqrt{3}$, so radius of the sphere is $r=\frac{d}{2}=2\sqrt{3}$

Volume of the cube will be $volume_{cube}=side^3=4^3=64$;
Volume of the sphere will be $volume_{sphere}=\frac{4}{3}\pi r^3=\frac{4}{3}\pi 24\sqrt{3}=32\sqrt{3}\pi$.

The ratio of the volume of the cube to the volume within the sphere that is not occupied by the cube is $\frac{64}{32\sqrt{3}\pi-64}$, and after some calculations you'll get that this ratio is closer to 10/17 than to any other ratios in answer choices.

Answer: A.

P.S. I really doubt that GMAT would offer such a problem, as it requires lengthy calculations and bad approximations (π, $\sqrt{3}$).

Discussed at: http://gmatclub.com/forum/hard-geometry-99533.html

33. Coordinate Geometry

On the coordinate plane (6, 2) and (0, 6) are the endpoints of the diagonal of a square. What is the distance between point (0, 0) and the closest vertex of the square?

A. 1/sqrt (2)
B. 1

C. sqrt (2)
D. sqrt (3)
E. 2*sqrt (3)

Given endpoints of diagonal of a square: B(0,6) and D(6,2). Let other vertices be A (closest to the origin, left bottom vertex) and C (farthest to the origin).

Length of the diagonal would be: $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(6-0)^2+(2-6)^2}=\sqrt{52}$

Coordinates of the midpoint M of the diagonal would be: $M(x,y)=(\frac{6+0}{2},\frac{2+6}{2})=(3,4)$.

Slope of the line segment AM*Slope of the line segment BD=-1 (as they are perpendicular to each other) --> $\frac{y-4}{x-3}*\frac{6-2}{0-6}=-1$ --> $\frac{y-4}{x-3}=\frac{3}{2}$ --> $y-4=\frac{3}{2}(x-3)$

Distance between the unknown vertices to the midpoint is half the diagonal:
$(x-3)^2+(y-4)^2=(\frac{\sqrt{52}}{2})^2=13$ --> $(x-3)^2+\frac{9}{4}(x-3)^2=13$ --> $(x-3)^2=4$ --> $x=1$ or $x=5$ --> $y=1$ or $y=7$

Hence the coordinates of the point A(1,1) and point C (5,7). Closest to the origin is A.
Distance $OA=\sqrt{2}$

Answer: C.

Discussed at: http://gmatclub.com/forum/coordinate-plane-90772.html

34. Word Problem
Eight litres are drawn off from a vessel full of water and substituted by pure milk. Again eight litres of the mixture are drawn off and substituted by pure milk. If the vessel now contains water and milk in the ratio 9:40, find the capacity of the vessel.
A. 21 litres
B. 22 litres
C. 20 litres
D. 14 litres
E. 28 litres

Let the capacity of the vessel be x.

After the first removal there would be $x-8$ liters of water left in the vessel. Note that the share of the water would be $\frac{x-8}{x}$;

After the second removal, the removed mixture of 8 liters will contain $8*\frac{x-8}{x}$ liters of water, so there will be $x-8-8*\frac{x-8}{x}=\frac{(x-8)^2}{x}$ liters of water left.

As the ratio of water to milk after that is $\frac{9}{40}$, then the ratio of water to the capacity of the vessel would be $\frac{9}{40+9}=\frac{9}{49}$.

So $\frac{\frac{(x-8)^2}{x}}{x}=\frac{9}{49}$ --> $\frac{(x-8)^2}{x^2}=\frac{9}{49}$ --> $\frac{x-8}{x}=\frac{3}{7}$ --> $x=14$.

Answer: D.

Discussed at: http://gmatclub.com/forum/eight-litres-are-drawn-off-from-a-vessel-full-of-water-and-s-95749.html

35. Inequality / Algebra
If $x>y^2>z^4$, which of the following statements could be true?
1. $x>y>z$
2. $z>y>x$
3. $x>z>y$

A..) 1 only
B. 1 & 2 only
C. 1 & 3 only
D. 2 & 3 only
E. 1,2 & 3

As this is a COULD be true question then even one set of numbers proving that statement holds true is enough to say that this statement should be part of correct answer choice.

Given: $x>y^2>z^4$.

1. $x>y>z$ --> the easiest one: if $x=100$, $y=2$ and $z=1$ --> this set satisfies $x>y^2>z^4$ as well as given statement $x>y>z$. So 1 COULD be true.

2. $z>y>x$ --> we have reverse order than in stem ($x>y^2>z^4$), so let's try fractions: if $x=\frac{1}{5}$, $y=\frac{1}{4}$ and $z=\frac{1}{3}$ then again the stem and this statement hold true. So 2 also COULD be true.

3. $x>z>y$ --> let's make x some big number, let's say 1,000. Next, let's try the fractions for z and y for the same reason as above (reverse order of y and z): $y=\frac{1}{3}$ and $z=\frac{1}{2}$. The stem and this statement hold true for this set of numbers. So 3 also COULD be true.

Answer: E.

Discussed at: http://gmatclub.com/forum/x-y-2-z-100465.html

36. Sequences
Series $A(n)$ is such that $i*A(i)=j*A(j)$ for any pair of positive integers (i,j). If $A(1)$ is a positive integer, which of the following is possible?

I. $2A(100)=A(99)+A(98)$

II. $A(1)$ is the only integer in the series

III. The series does not contain negative numbers

I only
II only

I and III only
II and III only
I, II, and III

A set of numbers $a_1, a_2, a_3, \ldots$ have the following properties: $i*a_i = j*a_j$ and $a_1 = positive\ integer$,
so $1*a_1 = 2*a_2 = 3*a_3 = 4*a_4 = 5*a_5 = ... = positive\ integer$.

We should determine whether the options given below can occur (note that the question is which **can be true, not must be true**).

I. $2a_{100} = a_{99} + a_{98}$ --> as $100a_{100} = 99a_{99} = 98a_{98}$, then $2a_{100} = \frac{100}{99}a_{100} + \frac{100}{98}a_{100}$ --> reduce by a_{100} --> $2 = \frac{100}{99} + \frac{100}{98}$ which is not true. Hence this option **cannot be true**.

II. a_1 is the only integer in the series. If $a_1 = 1$, then all other terms will be non-integers --> $a_1 = 1 = 2a_2 = 3a_3 = ...$ --> $a_2 = \frac{1}{2}$, $a_3 = \frac{1}{3}$, $a_4 = \frac{1}{4}$, and so on. Hence this option **can be true**.

III. The series does not contain negative numbers --> as given that $a_1 = positive\ integer = n*a_n$,
then $a_n = \frac{positive\ integer}{n} = positive\ number$, hence this option is **always true**.

Answer: D (II and III only).

Discussed at: http://gmatclub.com/forum/data-suffiency-tough-question-96538.html

37. Number Properties
If a and b are positive integers such that a/b = 2.86, which of the following must be a divisor of a?
A.1. 10
B. 13
C. 18
D. 26
E. 50

$\frac{a}{b} = 2.86 = \frac{286}{100} = \frac{143}{50}$ --> $b = \frac{50a}{143} = \frac{50a}{11*13}$, for b to be an integer a must have all the factors of 143 (50 has none of them). Hence a must be divisible by both 11 and 13.

Answer: B.

Discussed at: http://gmatclub.com/forum/prime-factor-100704.html

38. Rate problem
A boat travelled upstream 90 miles at an average speed of (v-3) miles per hour and then traveled the same distance downstream at an average speed of (v+3) miles per hour. If the trip upstream took a half hour longer than the trip downstream, then how many hours did it take the boat to travel downstream?
A. 2.5
B. 2.4
C. 2.3
D. 2.2
E. 2.1

Trip upstream took $\frac{90}{v-3}$ hours and trip downstream took $\frac{90}{v+3}$ hours. Also given that the difference in times was $\frac{1}{2}$ hours --> $\frac{90}{v-3}-\frac{90}{v+3}=\frac{1}{2}$;

$\frac{90}{v-3}-\frac{90}{v+3}=\frac{1}{2}$ --> $\frac{90(v+3)-90(v-3)}{v^2-9}=\frac{1}{2}$ --> $\frac{90*6}{v^2-9}=\frac{1}{2}$ --> $v^2=90*6*2+9$ --> $v^2=9*(10*6*2+1)$ --> $v^2=9*121$ --> $v=3*11=33$;

Trip downstream took $\frac{90}{v+3}=\frac{90}{33+3}=2.5$ hours.

Answer: A.

Discussed at: http://gmatclub.com/forum/difficult-ps-problem-help-100767.html

39. Modulus

If x<0, then $\sqrt{-x*|x|}$ equals:

A. $-x$
B. -1
C. 1
D. x
E. $\sqrt{x}$

Remember: $\sqrt{x^2}=|x|$.

The point here is that **square root function can not give negative result**: wich means that $\sqrt{some\ expression}\geq0$.

So $\sqrt{x^2}\geq0$. But what does $\sqrt{x^2}$ equal to?

Let's consider following examples:
If $x=5$ --> $\sqrt{x^2}=\sqrt{25}=5=x=positive$;
If $x=-5$ --> $\sqrt{x^2}=\sqrt{25}=5=-x=positive$.

So we got that:
$\sqrt{x^2}=x$, if $x\geq0$;
$\sqrt{x^2}=-x$, if $x<0$.

What function does exactly the same thing? The absolute value function! That is why $\sqrt{x^2}=|x|$

Back to the original question:
$\sqrt{-x*|x|}=\sqrt{(-x)*(-x)}=\sqrt{x^2}=|x|=-x$

Or just substitute the value let $x=-5<0$ --> $\sqrt{-x*|x|}=\sqrt{-(-5)*|-5|}=\sqrt{25}=5=-(-5)=-x$.

Answer: A.

Discussed at: http://gmatclub.com/forum/square-root-and-modulus-100303.html

41. Word Problem

A certain car averages 25 miles per gallon when driving in the city and 40 miles to the gallon when driving on the highway. According to these rates, which of the following is closest to the number of

miles per gallon that the car averages when it is driven 10 miles in the city and 50 miles on the highway?
A. 28
B. 30
C. 33
D. 36
E. 38

Car averages x miles per gallon, means that 1 gallon is enough to drive x miles.

We are asked to find average **miles per gallon** (miles/gallon) --> average miles per gallon would be **total miles driven** divided by **total gallons used**(miles/gallon).

Total miles driven is 10+50=60.

As car averages 25 miles per gallon in the city for 10 miles in the city it will use 10/25=0.4 gallons;
As car averages 40 miles per gallon on the highway for 50 miles on the highway it will use 50/40=1.25 gallons;

So average miles per gallon equals to $\frac{60}{0.4+1.25}\approx 36$

Answer: D.

Discussed at: http://gmatclub.com/forum/more-prep-questions-98942.html

42. Remainders
When 26854 and 27584 are divided by a certain two digit prime number, the remainder obtained is 47. Which of the following choices is a possible value of the divisor?
A. 61
B. 71
C. 73
D. 89

Given:

$27584=pq_1+47$
$26854=pq_2+47$
Where p is the prime number (divisor) and q_1 and q_2 are the quotients (integers ≥ 0).

Subtract equations: $27584-26854=pq_1+47-pq_2-47$ --> $730=2*5*73=p(q_1-q_2)$.
Product of two integers p, **two digit prime number**and (q_1-q_2) is $730=2*5*73$, as p is **two digit prime number**, it can be only 73 (2 and 5 are single digit).

Answer: 73.

Discussed at: http://gmatclub.com/forum/number-system-60282.html

43. Remainders
If x and y are positive integers and x/y has a remainder of 5, what is the smallest possible value of xy?

Given $x=qy+5$, where q is a quotient, an integer ≥ 0. Which means that the least value of x is when $q=0$, in that case $x=5$. This basically means that numerator x, is less than denominator y.

Now the smallest denominator y , which is more than numerator $x=5$ is 6. so we have $x=5$ and $y=6$ --> $xy=30$.

Discussed at: http://gmatclub.com/forum/remainder-of-89470.html

44. Remainders
When positive integer *n* is divided by 5, the remainder is 1. When *n* is divided by 7, the remainder is 3. What is the smallest positive integer *k* such that *k*+*n* is a multiple of 35?
A. 3
B. 4
C. 12
D. 32
E. 35

Positive integer n is divided by 5, the remainder is 1 --> $n=5q+1$, where q is the quotient --> 1, 6, 11, 16, 21, 26, **31**, ...
Positive integer n is divided by 7, the remainder is 3 --> $n=7p+3$, where p is the quotient --> 3, 10, 17, 24, **31**, ...

You can not use the same variable for quotients in both formulas, because quotient may not be the same upon division n by two different numbers.

For example 31/5, quotient q=6 but 31/7, quotient p=4.

There is a way to derive general formula for n (of a type $n=mx+r$, where x is divisor and r is a remainder) based on above two statements:

Divisor x would be the least common multiple of above two divisors 5 and 7, hence $x=35$.

Remainder r would be the first common integer in above two patterns, hence $r=31$.

Therefore general formula based on both statements is $n=35m+31$. Thus the smallest positive integer k such that k+n is a multiple of 35 is 4 --> $n+4=35k+31+4=35(k+1)$.

Answer: B.

Discussed at: http://gmatclub.com/forum/good-problem-90442.html

45. Statistics
A certain list has an average of 6 and a standard deviation of d (d is positive). Which of the following pairs of data when added to the list, must result in a list of 102 data with standard deviation less than d?
A. (-6;0)
B. (0;0)
C. (0;6)
D. (0;12)
E. (6;6)

"Standard deviation shows how much variation there is from the mean. A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values."

So when we add numbers, which are far from the mean we are stretching the set making SD bigger and when we add numbers which are close to the mean we are shrinking the set making SD smaller.

According to the above adding two numbers which are closest to the mean will shrink the set most, thus decreasing SD by the greatest amount.

Closest to the mean are 6 and 6 (actually these numbers equal to the mean) thus adding them will definitely shrink the set, thus decreasing SD.

Answer: E.

Discussed at: http://gmatclub.com/forum/standard-deviation-97473.html

46. Rate Problem
If a motorist had driven 1 hour longer on a certain day and at an average rate of 5 miles per hour faster, he would have covered 70 more miles than he actually did. How many more miles would he have covered than he actually did if he had driven 2 hours longer and at an average rate of 10 miles per hour faster on that day?
A. 100
B. 120
C. 140
D. 150
E. 160

Let t be the actual time and r be the actual rate.

"If a motorist had driven 1 hour longer on a certain day and at an average rate of 5 miles per hour faster, he would have covered 70 more miles than he actually did" --> $(t+1)(r+5)-70=tr$ --> $tr+5t+r+5-70=tr$ --> $5t+r=65$;

"How many more miles would he have covered than he actually did if he had driven 2 hours longer and at an average rate of 10 miles per hour faster on that day?" --> $(t+2)(r+10)-x=tr$ --> $tr+10t+2r+20-x=tr$ --> $2(5t+r)+20=x$ --> as from above $5t+r=65$, then $2(5t+r)+20=2*65+20=150=x$ --> so $x=150$.

Answer: D.

OR another way:

70 miles of surplus in distance is composed of driving **at 5 miles per hour faster for t hours** *plus* **driving for $r+5$ miles per hour for additional 1 hour** --> $70=5t+(r+5)*1$ --> $5t+r=65$;

With the same logic, surplus in distance generated by driving **at 10 miles per hour faster for 2 hours longer** will be composed of driving **at 10 miles per hour faster for t hours** *plus* **driving for $r+10$ miles per hour for additional 2 hour** --> $surplus=x=10t+(r+10)*2$ --> $x=2(5t+r)+20$ --> as from above $5t+r=65$, then $x=2(5t+r)+20=150$.

Answer: D.

Discussed at: http://gmatclub.com/forum/ps-motorist-61729.html

47. Fractions
The ratio of two positive numbers is 3 to 4. If k is added to each number the new ratio will be 4 to 5, and the sum of the numbers will be 117. What is the value of k?
A. 1
B. 13
C. 14
D. 18
E. 21

The ratio of two positive numbers is 3 to 4 --> $\frac{a}{b}=\frac{3x}{4x}$, for some positive integer x.

If k is added to each number the new ratio will be 4 to 5 --> $\frac{3x+k}{4x+k}=\frac{4}{5}$ --> $15x+5k=16x+4k$ --> $x=k$.

The sum of the numbers will be 117 (I believe it means that the sum of the numbers after we add k to each) --> $3x+k+4x+k=117$ --> $7x+2k=117$ --> as from above $x=k$ --> $7k+2k=117$ --> $k=13$.

Answer: B.

Discussed at: http://gmatclub.com/forum/interesting-ratio-problem-95198.html

48. Word Problem / Percent
Before being simplified, the instructions for computing income tax in country R were to add 2 percent of one's annual income to the average (Arithmetic mean) of 100 units of country R's currency and 1 percent of one's annual income. Which of the following represents the simplified formula for computing the income tax, in country R's currency, for a person in that country whose annual income is I.
A. 50 + (I/200)
B. 50 + (3I/100)
C. 50 + (I/40)
D. 100 + (I/50)
E. 100 + (3I/100)

Tax is the sum of the following:

2 percent of one's annual income - $0.02I$;

The average (arithmetic mean) of 100 units of country R's currency and 1 percent of one's annual income - $\frac{100+0.01I}{2}$.

$$Tax=0.02*I+\frac{100+0.01*I}{2}=50+\frac{0.04*I+0.01*I}{2}=50+\frac{0.05*I}{2}=50+\frac{I}{40}.$$

Answer: C.

Discussed at: http://gmatclub.com/forum/ps-from-gmat-prep-91752.html

49. Rate Problem
Circular gears P and Q start rotating at the same time at constant speeds. Gear P makes 10 revolutions per minute, and gear Q makes 40 revolutions per minute. How many seconds after the gears start rotating will gear Q have made exactly 6 more revolutions than gear P?
A. 6
B. 8
C. 10
D. 12
E. 15

Note that we are given revolutions per minute and asked about revolutions in seconds. So we should transform per minute to per second.

Gear P makes 10 revolutions per minute --> gear P makes 10/60 revolutions per second;
Gear Q makes 40 revolutions per minute --> gear Q makes 40/60 revolutions per second.

Let t be the time in seconds needed for Q to make exactly 6 more revolutions than gear P --

> $\frac{10}{60}t+6=\frac{40}{60}t$ --> $t=12$.

Answer: D.

Discussed at: http://gmatclub.com/forum/gmat-prep-question-99043.html

50. If k is an integer, and 35^2-1/k is an integer, then k could be each of the following, EXCEPT
A. 8
B. 9
C. 12
D. 16
E. 17

$$\frac{35^2-1}{k}=\frac{(35-1)(35+1)}{k}=\frac{34*36}{k}=\frac{2^3*3^2*17}{k}$$

From the answer choices only $16=2^4$ is not a factor of numerator, hence in case $k=16$, $\frac{35^2-1}{k}$ won't be an integer, hence k can not be 16.

Answer: D.

Discussed at: http://gmatclub.com/forum/35-2-1-k-94473.html